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Dipolar interactions in the dilute antiferromagnet in a uniform field

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Abstract. We examine the role played by dipolar interactions in the equilibrium critical behaviour of the dilute antiferromagnet in a uniform field (DAFF) The DAFF has been known to map onto a random field model, and we perform a similar mapping, only this time including dipolar interactions. We conclude that dipolar forces affect the DAFF in a way no different from the case for the corresponding pure system Dilution does not generate any additional long-range forces or other effects arising from dipolar forces In particular, in the final random field model which the DAFF maps onto, we find that the mean-field propagator is unaffected by the inclusion of dipolar interactions save for some constant terms We restrict our work to the critical region close to the transition temperature and assume a domain-free state.

1. Introduction

The dilute antiferromagnet in a uniform field (DAFF) has been shown to be described by a random field model (Fishman and Aharony [1], Cardy [2]); it is the dilution of bonds or sites, coupled to the uniform field, that generates the random field [1, 2]. Taking into account nearest-neighbour interactions only, Fishman and Aharony [1] and Cardy [2] showed that the Hamiltonian describing the DAFF is the same as that describing the random field Ising model (RFIM). However, all magnetic systems contain long-range anisotropic dipolar forces. These forces decisively affect the universality properties of certain systems, e.g. the Ising ferromagnet [3]. The influence of dipolar forces on the DAFF has been treated briefly by Nattermann [4]; he showed that in the low-temperature multidomain state, in the presence of dipolar forces the DAFF is no longer in the same universality class as the random field Ising model with nearest-neighbour interactions only (short range RFIM). Nattermann [4] also predicted that the upper critical dimension d_c still remains at 6 for both systems even if one takes into account dipolar forces in the DAFF, and high dimensions close to d_c, dipolar interactions will not change the *static* critical exponents. However, in Nattermann's work the ferromagnetic fluctuations were neglected [5] and it is the purpose of this paper to include them carefully

The influence of dipolar interactions on a pure antiferromagnet has been studied by Gehring [6], starting with both antiferromagnetic and ferromagnetic order parameters and then integrating out the ferromagnetic order parameter. The resultant

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Hamiltonian was the same as that for the pure Ising model, except that the quartic ϕ^4 term has an angular contribution due to dipolar forces. The quadratic ϕ^2 term is unaffected. This work seeks to investigate if any additional dipolar effects would arise with diluting the antiferromagnet, as in this case the ferromagnetic fluctuations couple directly to the antiferromagnetic order parameter [1, 2], as well as to the antiferromagnetic energy density as for the pure metamagnet. With dilution we have a random field model [1], and we can expect the non-equilibrium behaviour typical of such systems (e.g. see Villain [7]). To study the dynamics plus the effects of dipolar forces on the dynamics, one may have to go beyond deriving the Hamiltonian.

This work will restrict itself mostly to the equilibrium situation, though a brief mention of dipolar effects on the non-equilibrium behaviour will be made. As such we will work within the following restrictions:

(i) extremely close to the transition temperature T_N
(ii) high dimensions close to the upper critical dimension d_c
(iii) the transition is to an antiferromagnetically ordered phase.

2. Derivation of the Hamiltonian

Using field-theoretical methods, Cardy [2] mapped the DAFF onto the RFIM, but he did not include dipolar interactions. Nelson and Fisher [8] treated the problem of a pure metamagnet in a uniform field, using renormalization-group (RG) scaling arguments and two order parameters—an antiferromagnetic primary and a ferromagnetic secondary order parameter—integrating out the secondary order parameters. They too did not take into account dipolar forces. Our approach will combine the methods of Cardy [2] and Nelson and Fisher [8].

The Hamiltonian in real space for the site-diluted Ising antiferromagnet in a uniform field is as follows

$$\mathscr{H}=\sum_{rr'} J(r-r')\varepsilon(r)\varepsilon(r')S(r)S(r')-H\sum_{r}\varepsilon(r)S(r) \tag{1}$$

where

$$S(r)=\pm 1$$

$$\varepsilon(r)=\begin{cases}0 & \text{if a spin is absent at site } r\\ 1 & \text{if a spin is present at site } r\end{cases}$$

$$x=\langle\varepsilon(r)\rangle$$

$$\varepsilon(r)=x+\rho(r)$$

and H is the uniform field. $J(r-r')$ is the interaction between spins at r and r', and includes nearest-neighbour exchange as well as long range dipolar interactions.

We use the Hubbard–Stratanovitch transformation to convert to a field theory and shift to eliminate the uniform magnetization M_0. The magnetization fluctuation terms are expressed in terms of a ferromagnetic and an antiferromagnetic component $\sigma(q)$ and $\phi(q)$ respectively in a reduced zone which is defined by k_0 such that

$$e^{ik_0\cdot R_n}=\begin{cases}+1 & \text{site } n \text{ on sublattice } A\\ -1 & \text{site } n \text{ on sublattice } B.\end{cases}$$

We define the Green functions for a pure ferromagnet $f(q)$ and the Green functions $G(q_1, q_2)$ and $K(q_1, q_2)$ for the dilute ferromagnet and antiferromagnet respectively. For convenience we use a tensor notation. Repeated indices are *not* summed over, unless explicitly stated.

$$f(q_i)=f_i \qquad G(q_i, q_j)=G_{ij} \qquad J(k_0+q_i)=J_{0+i}$$
$$f_i^{-1}=\frac{k_B T}{2}J_i-\lambda_2 x J_i J_{-i} \tag{2}$$

$$G_{ij}^{-1}=f_i^{-1}\delta_{i+j}-\lambda_2\rho_{-i-j}J_iJ_j \tag{3}$$

$$K_{ij}^{-1}=f_{0+i}^{-1}\delta_{i+j}-\lambda_2\rho_{-i-j}J_{0+i}J_{0+j} \tag{4}$$

where

$$\lambda_2=\tfrac{1}{2}[1-(M_0\bar{J})^2] \tag{5}$$

and

$$\bar{J}=\sum_r J(r'-r). \tag{6}$$

The non-analytic dipolar contribution is present in J_q and hence in G_{ij} and f_i. The relevant terms in the effective Hamiltonian are given below.

$$\mathcal{H}_3=\sum_{ij}K_{ij}^{-1}\phi_i\phi_j+P\sum_i\rho_{0+i}J_{0-i}\phi_i+\tfrac{1}{3}M_0\bar{J}\sum_{ijl}\rho_{i+j+l+0}J_{0+i}J_{0+j}J_{0+l}\phi_i\phi_j\phi_l$$
$$+\frac{x}{12}\sum_{ijl}J_{0+i}J_{0+j}J_{0+l}J_{0+i+j+l}\phi_i\phi_j\phi_l\phi_{i+j+l}+\sum_{ij}G_{ij}^{-1}J_iJ_j\sigma_i\sigma_j+2\sum_i B_iJ_i\sigma_i \tag{7}$$

where

$$P=[\tfrac{1}{3}(M_0\bar{J})^2-1]M_0\bar{J} \tag{8}$$

and

$$B_i[\phi]=\frac{P}{2}\rho_i+\tfrac{1}{2}\lambda_2\sum_j\rho_{0+i+j}J_{0+j}\phi_j+\tfrac{1}{2}M_0\bar{J}\sum_{jl}\varepsilon_{i+j+l}J_{0+j}J_{0+l}\phi_j\phi_l. \tag{9}$$

We complete the square in σ and express the Hamiltonian entirely in terms of ϕ:

$$\mathcal{H}_3=\sum_{ij}K_{ij}^{-1}\phi_i\phi_j+P\sum_i\rho_{0+i}J_{0+i}\phi_i+\tfrac{1}{3}M_0\bar{J}\sum_{ijl}\rho_{0+i+j+l}J_{0+i}J_{0+j}J_{0+l}\phi_i\phi_j\phi_l$$
$$+\frac{x}{12}\sum_{ijl}J_{0+i}J_{0+j}J_{0+l}J_{0+i+j+l}\phi_i\phi_j\phi_l\phi_{i+j+l}-\sum_{ij}B_i\Gamma_{ij}B_j \tag{10}$$

where we have defined

$$\Gamma_{ij}\equiv J_iG_{ij}J_j. \tag{11}$$

We proceed to determine this term $B_i\Gamma_{ij}B_j$, which contains ϕ terms and no σ terms.

$$B_i\Gamma_{ij}B_j=\left(\frac{P}{2}\right)^2\rho_i\Gamma_{ij}\rho_j-\frac{P\lambda_2}{2}\sum_a\rho_i\Gamma_{ij}\rho_{j+a+0}J_{a+0}\phi_a$$
$$+\lambda_2^2\sum_{ab}\phi_aJ_{a+0}\rho_{0+a+i}\Gamma_{ij}\rho_{j+b+0}J_{0+b}\phi_b+\frac{P}{4}(M_0\bar{J})^2\sum_{ab}\rho_i\Gamma_{ij}\varepsilon_{j-i+b}J_{a+0}J_{a+0}\phi_a\phi_b$$
$$-\tfrac{1}{2}(M_0\bar{J})^2\lambda_2\sum_{acd}\phi_aJ_{a+0}\rho_{0+a+i}\Gamma_{ij}\rho_{j+c+d}J_{c+0}J_{d+0}\phi_c\phi_d$$
$$+\tfrac{1}{4}(M_0\bar{J})^4\sum_{abcd}\phi_a\phi_bJ_{a+0}J_{b+0}\varepsilon_{a-b+i}\Gamma_{ij}\varepsilon_{j+c+d}J_{c+0}J_{d+0}\phi_c\phi_d. \tag{12}$$

We see that after integrating out σ, there are contributions to ϕ, ϕ^2, ϕ^3, ϕ^4.

3. Analysis of the Hamiltonian

We now analyse the Hamiltonian of equation (10) and the terms in equation (12). The ferromagnetic order parameter was non-critical and mean-field; integrating it out of the partition function could be thought of as averaging over the ferromagnetic interactions in the DAFF, from which additional contributions to the antiferromagnetic terms arise. Gehring [6] performed a similar 'integrating out' for the corresponding non-dilute system; there dipolar interactions were taken into account, and she found that there was only a contribution to the quartic ϕ^4 term. We analyse the role of both random dilution and dipolar interactions in these contributions for the DAFF. Γ_{ij} is related to the dilute ferromagnetic propagator and contains a non-analytic dipolar contribution. From equation (12) it is obvious that all terms in $B_i\Gamma_{ij}B_j$ contain non-analytic dipolar contributions. However, all terms also contain the random dilution variables ρ, ε, and it remains to be determined whether the *ensemble average* of each term will still contain any non-analytic contribution. We will examine the final form of each power of ϕ in the Hamiltonian of equation (11) separately.

To determine the ensemble average of all terms, we use the following relations:

$$\overline{\rho_{a+b}\rho_{c+d}}=\overline{\rho_{a+b+0}\rho_{c+d+0}}=x(1-x)\delta_{a+b+c+d} \tag{13a}$$
$$\overline{\rho_{a+b}\rho_{c+d+0}}=0. \tag{13b}$$

($\bar{A}$ means the ensemble average of A.)

We begin with the quadratic ϕ^2 term, because the inverse of the quadratic term is the mean-field susceptibility. If the mean-field susceptibility has contributions from the 'integrating out', then the system will be affected at the mean-field level. Higher order terms may affect the universality class and critical behaviour of the system, but only at a level beyond the mean-field approximation. In the standard graphical perturbation treatment of critical systems beyond the mean-field approximation, all perturbation terms are expressed in terms of the mean-field propagator. If the mean-field propagator contains any contribution terms that changes the physics in an important way, all terms in the perturbation series will be correspondingly affected. We wish to determine if the *ensemble-averaged* quadratic mean-field propagator contains any long-range force or anisotropy arising from the combined effects of dilution and dipolar interactions. If the ensemble-averaged quadratic term in the DAFF contains any anisotropic contribution, the universality of the system will be altered and the static critical exponents affected [3]. By comparison, for the non-dilute antiferromagnet in a uniform field, the quadratic propagator is unaffected by the inclusion of dipolar interactions after integrating out the σ [6].

The full quadratic term in the final Hamiltonian is $C^{(2)}_{ij}\phi_i\phi_j$ where the coefficient

$$C^{(2)}_{ij}=K^{-1}_{ij}+\left[\lambda_2^2\sum_{ab}\rho_{0+i+a}\Gamma_{ab}\rho_{b+j+0}+\tfrac{1}{4}P(M_0\bar{J})^2\sum_{ab}\rho_a\Gamma_{ab}\varepsilon_{b+i+j}\right]J_{0+i}J_{0+j}. \tag{14}$$

We wish to determine the ensemble-averaged quadratic term, which will involve $\overline{C^{(2)}}$. This in turn involves Γ_{ij} and its ensemble average $\overline{\Gamma_{ij}}$. From equation (11), we need to determine G_{ij} which is the inverse of

$$G^{-1}_{ij}=f_i^{-1}\delta_{i+j}-\lambda_2\rho_{i+j}J_iJ_j. \tag{15}$$

We define

$$g_i\equiv J_if_iJ_i \tag{16}$$

$$G_{ij}=f_i\delta_{i+j}+\lambda_2f_iJ_i\rho_{i+j}J_jf_j+\lambda_2^2\sum_b f_iJ_i\rho_{i+b}J_bf_bJ_b\rho_{b+j}J_jf_j+ \tag{17}$$

$$\Rightarrow\Gamma_{ij}=g_i\delta_{i+j}+\lambda_2g_i\rho_{i+j}g_j+\lambda_2^2\sum_b g_i\rho_{i+b}g_b\rho_{b+j}g_j+\dots \tag{18}$$

In deriving equation (18) we have used equations (11) and (16).

$$\overline{\Gamma_{ij}}=\left[\frac{k_BT}{2}J_i^{-1}-x\lambda_2-\Sigma_i\right]^{-1}\delta_{i+j}\equiv\gamma_i\delta_{i+j}. \tag{19}$$

Σ_i, or in ordinary momentum-space notation $\Sigma(q)$, is the self-energy and is analytic and well defined even as q tends to zero. For a more detailed proof of equations (15)–(19) and the properties of Σ and γ, see the appendix. We have just derived a side result for spin waves in a dilute ferromagnet, even though this work is concerned with the dilute antiferromagnet. The effect of dilution is to create a self-energy contribution $\Sigma(q)$ to the energy of the ferromagnetic spin waves (Edwards and Jones [9]). By showing that dipolar interactions do not contribute any non-analytic term to the self-energy, we have demonstrated that dilution does not create any dipolar or higher order long range contribution to the spin wave energy and ensemble-averaged Green function in the ferromagnet:

$$\sum_{ab}\overline{\rho_{0+i+a}\Gamma_{ab}\rho_{b+j+0}}=x(1-x)\left(\sum_a\gamma_a\right)\delta_{i+j} \tag{20}$$

$$\sum_{ab}\overline{\rho_a\Gamma_{ab}\varepsilon_{b+i+j}}=\left(\frac{1}{\lambda_2}\right)^2\Sigma(0)\delta_{i+j}. \tag{21}$$

For a more detailed proof of equations (20) and (21), refer to the appendix. In equation (20), γ_a is summed over the internal dummy momentum q_a; hence it is a constant independent of external momenta. $\Sigma(q)$ has a well defined limit at $q=0$; hence in equation (21) $\Sigma(0)$ is also a constant. Using equations (20), (21) the *ensemble-averaged* quadratic term is

$$\overline{C^{(2)}_{ij}\phi_i\phi_j}=\left[\frac{k_BT}{2}J_{0+i}-x\lambda_2J_{0+i}J_{0-i}-x(1-x)\lambda_2^2\left(\sum_a\gamma_a\right)J_{0+i}J_{0-i}\right.$$
$$\left.-\frac{P}{4}\left(\frac{M_0\bar{J}}{\lambda_2}\right)^2\Sigma(0)J_{0+i}J_{0-i}\right]\phi_i\phi_{-i}. \tag{22}$$

The self-energy $\Sigma(q)$ does not contain any dipolar contribution in the limit $q\to0$.

The ensemble averaged quadratic term is dependent on one momentum q_i, thus in real space it is translation-invariant. It contains no angular or non-analytic terms whatsoever; dipolar contributions are ensemble-averaged to constants in the quadratic term—this means in real space that no long-range forces exist. The effect of integrating out σ in the ensemble-averaged quadratic term merely gives rise to constant contributions, which renormalize the transition temperature T_N.

We now examine the random field term. The ϕ term in the final Hamiltonian is $C_i^1\phi_i$, where the linear coefficient

$$C_i^1 \equiv \left[P\rho_{0+i} + \frac{P}{2}\lambda_2 \sum_{ab} \rho_a \Gamma_{ab}\rho_{b+i+0}\right] J_{0+i}. \tag{23}$$

Using the relations (13), one can show $\overline{C^1}=0$, i.e. the linear coefficient C^1 is still a random field. However, the random field distribution is altered by the combined effects of dilution and long range interactions.

We now examine the higher order terms. The full cubic term in the final Hamiltonian is $C_{ijl}^{(3)}\phi_i\phi_j\phi_l$ where the coefficient

$$C_{ijl}^{(3)} = \left[\tfrac{1}{3}M_0\bar{J}\rho_{0+i+j+l} + \frac{\lambda_2}{2}(M_0\bar{J})^2 \sum_{ab} \rho_{0+i+a}\Gamma_{ab}\rho_{b+j+l}\right] J_{0+i}J_{0+j}J_{0+l}. \tag{24}$$

Using relations (13*a*), (13*b*), one can show that the ensemble average of the cubic term is zero, but there is a ϕ^6 contribution owing to the fact that the ensemble average of the square of the cubic term is non-zero. This will affect the tricritical behaviour when the quartic ϕ^4 term vanishes.

The quartic ϕ^4 contribution that arises from integrating out σ is

$$\tfrac{1}{4}(M_0\bar{J})^4 \sum_{ab} \phi_i\phi_j J_{i+0}J_{j+0}\varepsilon_{i+j+a}\Gamma_{ab}\varepsilon_{b+l+m}J_{l+0}J_{m+0}\phi_l\phi_m.$$

We evaluate the coefficient as follows,

$$\sum_{ab} \varepsilon_{i+j+a}\Gamma_{ab}\varepsilon_{b+l+m} = x^2 \sum_{ab} \delta_{i+j+a}\delta_{b+l+m}\Gamma_{ab} + \sum_{ab} \rho_{i+j+a}\Gamma_{ab}\rho_{b+l+m}. \tag{25}$$

From the appendix, using (A3), (A4), (A10), one can show

$$\sum_{ab} \overline{\varepsilon_{i+j+a}\Gamma_{ab}\varepsilon_{b+l+m}} = x^2[g_{i+j}^{-1} - \Sigma_{i+j}]^{-1}\delta_{i+j+l+m} + \left(\frac{1}{\lambda_2}\right)^2 \Sigma_{i+j}\left[1 + \frac{\Sigma_{i+j}}{g_{i+j}^{-1} - \Sigma_{i+j}}\right]\delta_{i+j+l+m}. \tag{26}$$

The ensemble average of this quartic ϕ^4 contribution has a non-analytic component $[g_{i+j}^{-1} - \Sigma_{i+j}]^{-1}$. Recall that g contains the non-analytic dipolar term even though Σ is analytic. This is very similar to the non-analytic quartic ϕ^4 contribution which Gehring [6] obtained from integrating out σ for the *pure* metamagnet in a uniform field. In Gehring's work [6], the non-analytic contribution is g_{i+j}. The quartic term in our final Hamiltonian is essentially the same as Gehring's [6] except for some constants.

One can also show the following:

$$\overline{C^{(3)}C^1} \neq 0 \tag{27a}$$

$$\overline{C^{(2)}C^{(2)}} \neq 0 \tag{27b}$$

$$\overline{C^{(2)}C^1C^1} \neq 0. \tag{27c}$$

Equations (27) show that there will be contributions to the quartic term from averaging various combinations of quadratic, cubic and random field terms. The final form of the quartic term will differ from that of Gehring [6], but should not change the physics in any important way.

4. Conclusion

We have analysed in mathematical detail the effects of dipolar interactions on the DAFF. Apart from some constants, the mean-field propagator of the DAFF and the four-point quartic coupling constant are essentially the same as those corresponding to the pure metamagnet in a uniform field with dipolar interactions investigated by Gehring [6]. In particular, the mean-field propagator is analytic and contains no long-range forces of any order. This is equivalent to saying that, in the mean-field approximation, dipolar forces do not contribute long-range forces of any order, being cancelled out by the oppositely aligned spins of the antiferromagnetic ordered state. Random dilution generates random patches of net ferromagnetism in the DAFF, yet any dipolar or higher order forces cancel out after averaging over the randomness. As a side result, we have shown that concentration fluctuations do not create any dipolar or long range contribution to the ensemble-averaged Green function in the *ferromagnet*. The quartic four-point coupling constant contains a non-analytic contribution that is essentially the same as that of the corresponding pure system apart from some constants Dilution creates a cubic term that is absent in the pure system, but the ensemble average of this term is zero. If we stay in the realm of a second-order phase transition, then the cubic term is irrelevant. We can control the uniform field so that the random field is weak and thus remain within a second-order phase transition. If we increase the uniform field, the random field strength increases and the system may reach a tricritical point, where the cubic term will be important; the square of the cubic term will give a ϕ^6 contribution. With nearest-neighbour interactions only, site dilution in the DAFF coupled to a uniform field generates a random field with a discrete distribution of a few points. Dipolar forces create a long-range effective field which renders the random field distribution continuous.

We have confined ourselves to the equilibrium critical properties of the DAFF. It is well known that the dynamic properties of random field models including the DAFF are very different from those of pure systems, involving different relaxation mechanisms and long relaxation times. Many actual random field models find it difficult to reach equilibrium for static exponents to be measured. The dynamics of random field models is still a topic of active controversial study (for a review, see Villain [7]). The DAFF is a random field model where it is experimentally possible to obtain an equilibrium state below the transition temperature. By cooling at zero uniform field and then subsequently applying a uniform field, an experimentalist can achieve the equilibrium long-range ordered state. For a review of experiments on random field systems, see Birgeneau *et al* [10] and Belanger [11].

We have considered the equilibrium thermodynamics which imply that the system evolves into a state of long-range order without domains in obtaining our results. Most random field systems reach a metastable state below the transition temperature. For the DAFF, it is possible to attain this metastable state by cooling at a non-zero uniform field. In the metastable state, domains exist, and Nattermann [4] has shown the dipolar forces favour this metastable domain state for the DAFF. This would imply that dipolar forces further discourage the system from jumping out of metastable barriers and hence lengthen the critical slowing down times. Along certain orientations, depending on the crystal symmetry, the antiferromagnet can have a domain wall that has a magnetic moment. To our knowledge, no work has been done on the potential influence dipolar forces can have on the domain wall dynamics, although Nattermann [4] has showed that dipolar forces are irrelevant in flat interfaces in the DAFF. Our guess is that whatever dipolar energy is stored in the domain walls will be small in comparison to the dipolar energy in the bulk domains. Further work should be done on this area.

Acknowledgment

The authors are indebted to Professor T Nattermann for invaluable discussions.

Appendix

To derive G_{ij} we follow the paper by Edwards and Jones [9], who expanded G in powers of f and ρ (cf equations (17) and (18)):

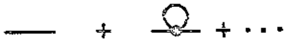
$$G_{ij}=f_i\delta_{i+j}+\lambda_2 f_i J_i\rho_{i+j}J_j f_j+\lambda_2^2\sum_b f_i J_i\rho_{i+b}J_b f_b J_b\rho_{b+j}J_j f_j+\ldots \tag{A1}$$

$$\Rightarrow\Gamma_{ij}=g_i\delta_{i+j}+\lambda_2 g_i\rho_{i+j}g_j+\lambda_2^2\sum_b g_i\rho_{i+b}g_b\rho_{b+j}g_j+\ldots. \tag{A2}$$

They then utilized a diagrammatic expansion:

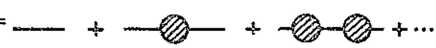
$\Gamma_{ij}=$ 

where ——— $=g(q)$ and - - - $=\lambda_2\rho(q)$

$\overline{\Gamma_{ij}}=$ 

where $+=\lambda_2^2x(1-x)$ and

 $=\lambda_2^2x(1-x)\left(\int g(q)\,\mathrm{d}q\right)g^2(q_i)$

$\overline{\Gamma_{ij}}=$ 

 is the self-energy $\Sigma(q)$, which is the sum of all irreducible diagrams without external propagators or 'legs'. Irreducibility is defined as the impossibility of cutting one connected diagram into two separate diagrams by cutting just one propagator.

$$\overline{\Gamma_{ij}}=\gamma_i\delta_{i+j} \tag{A3}$$

$$\Rightarrow\gamma_i=g_i+g_i\Sigma_i g_i+g_i\Sigma_i g_i\Sigma_i g_i+\ldots=[g_i^{-1}-\Sigma_i]^{-1} \tag{A4}$$

(A4) is the Dyson equation. $g(q)$ contains the non-analytic dipolar contribution $(q_z/q)^2$. $\Sigma(q)$ is analytic and well defined in the limit $q\to 0$. $\Sigma(q)$ contains integrals whose dipolar angular terms are dominated by the internal dummy momenta as the external momentum q goes to zero, e.g.

$$\lim_{q\to 0}\left(\frac{q_{1z}+q_z}{|q_1+q|}\right)^2=\left(\frac{q_{1z}}{q_1}\right)^2.$$

Such angular terms when summed over all directions and angles for the internal momenta are well defined and independent of the direction of the external momentum q.

Having determined Γ_{ij}, we can now calculate equations (20), (21). We utilize relations (13). From equation (13*b*) it is not difficult to prove that

$$\overline{\rho_{0+i}\rho_1\rho_2 \cdots \rho_n} = 0. \tag{A5}$$

Γ_{ab} contains ρ_i terms and no 'staggered' dilution terms like ρ_{0+i}. Using (A5), we can perform the separation

$$\overline{\rho_{0+i+a}\Gamma_{ab}\rho_{b+j+0}} = \overline{\rho_{0+i+a}\rho_{b+j+0}}\,\overline{\Gamma_{ab}}$$

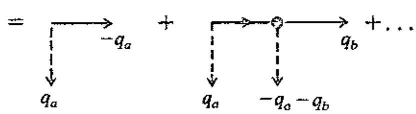
$$\Rightarrow \overline{\rho_{0+i+a}\Gamma_{ab}\rho_{b+j+0}} = x(1-x)\delta_{i+a+b+j}(\gamma_a\delta_{a+b})$$

which leads to equation (20).

$$\rho_a\Gamma_{ab}\varepsilon_{b+i+j} = x\rho_a\Gamma_{ab}\delta_{b+i+j} + \rho_a\Gamma_{ab}\rho_{b+i+j}. \tag{A6}$$

We begin with $\rho_a\Gamma_{ab}$. From (A2),

$$\rho_a\Gamma_{ab} = \rho_a[g_a\delta_{a+b} + \lambda_2 g_a\rho_{a+b}g_b + \ldots]$$



(A7)

On averaging,

$$\overline{\rho_a\Gamma_{ab}} = \Big[$$


$$\Big] = \frac{\Sigma_b}{\lambda_2}[g_b^{-1} - \Sigma_b]^{-1}\delta_b. \tag{A8}$$

By conservation of momentum, because there is only one external momentum, we require $q_a = 0$. $g(q)$ is defined only for $q \neq 0$. This is because $\lim_{q\to 0} g(q)$ is not well defined; at $q = 0$, we get a shape-dependent constant term which has already been taken into account in the constant magnetization M_0 [3]. By definition, $g(q) \equiv 0$ at $q = 0$ [3]. It follows that

$$\overline{\rho_a\Gamma_{ab}} = 0. \tag{A9}$$

We now turn our attention to $\rho_a\Gamma_{ab}\rho_{b+i+j}$.

From (A1),

$$\sum_{ab} \rho_a\Gamma_{ab}\rho_{b+i+j} = \sum_{ab} \rho_a g_a \rho_{a+i+j} + \sum_{ab} \lambda_2\rho_a g_a\rho_{a+b}g_b\rho_{b+i+j} + \ldots \tag{A10}$$

$$\sum_{ab} \overline{\rho_a\Gamma_{ab}\rho_{b+i+j}}$$

$$= \left(\frac{1}{\lambda_2}\right)^2 [\Sigma(0) + \Sigma(0)g(0)\Sigma(0) + \Sigma(0)g(0)\Sigma(0)g(0)\Sigma(0) + \ldots]\delta_{i+j}. \tag{A11}$$

But $g(0) \equiv 0$ [3]. Combining (A9) and (A11), we obtain

$$\sum_{ab} \overline{\rho_a\Gamma_{ab}\varepsilon_{b+i+j}} = \left(\frac{1}{\lambda_2}\right)^2 \Sigma(0)\delta_{i+j} \tag{A12}$$

which is equation (21).

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